NOISE IN RECEIVING AERIAL SYSTEMS

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ABSTRACT. Several authors have investigated theoretically and experimentally the signal/noise ratio in receiving aerial systems, but the problem of the validity of Nyquist's theorem for radiation resistance has hitherto not been satisfactorily resolved.

The problem is here discussed, and it is concluded that (i) for an aerial in an enclosure at uniform temperature, the radiation resistance is at that temperature from the point of view of noise, and thus (ii), for an aerial in free space, no noise originates in the radiation resistance. These results are shown to be consistent with the Rayleigh-Jeans radiation law, while the quantum-theory form of Nyquist's equation leads to the Planck radiation law.

The estimation of the noise occurring in practical aerial systems is discussed, consideration being given to the various external noise fields, viz. thermal radiation, Jansky noise, atmospherics. The actual noise received may be expressed conveniently by the equivalent temperature T_r of the radiation resistance. The general problem of evaluating the signal/noise ratio for any values of received circuit- and valve-noise is analysed and a criterion (K) of efficiency of an aerial system is deduced. The paper concludes with a numerical calculation of the performance of (1) a vertical aerial inductively coupled to a tuned-grid circuit and (2) a tuned-loop aerial, which are typical examples of an efficient and an inefficient system respectively.

§ 1. INTRODUCTION

In computing the signal/noise ratio in receiving systems, it is necessary to estimate the noise arising in the aerial circuit. Thermal noise is given by Nyquist's theorem, which states that the mean square fluctuation e.m.f. in the frequency band ν to $\nu + d\nu$ appearing in an impedance $Z(=R+jX)$ is given by

$$d\overline{e^2} = 4\boldsymbol{k}TR \,.\, d\nu, \qquad \ldots\ldots (1)$$

where $\boldsymbol{k}$ = Boltzmann's constant ($1{\cdot}37 \times 10^{-16}$ erg/deg.) (Nyquist, 1928). This formula directly associates the thermal agitation e.m.f. with the resistance, and the temperature T concerned is that of the resistive component of the impedance only.

Nyquist's proof of his theorem is a thermodynamic one, and is hence quite general and does not refer to the processes involved. However, Bernamont (1937) and Bakker and Heller (1939) have derived " mechanistic " proofs of the theorem by applying both classical and quantum statistics to the thermal agitation of free electrons in a conductor.

Moullin (1938) concludes on experimental grounds that " any dissipative element in a network is the source and initiation of thermal fluctuation voltage ". It seems certain, however, that this statement was not intended to apply to radiation resistance, for the latter, not being directly associated with any molecular dissipative process, requires separate consideration. Since the radiation resistance of an aerial is unlike other forms of resistance, e.g. it is independent of the temperature of the material of the aerial, and it does not involve any Brownian motion (as does ohmic resistance), it might be anticipated that radiation resistance is not of itself a source of fluctuation noise.

In this paper, however, it is shown that radiation resistance can apparently be the source of thermal noise, and it obeys Nyquist's theorem when the aerial is in radiative equilibrium with its surroundings.

§2. REVIEW OF PREVIOUS WORK

Previous literature contains only a few direct references to the validity of Nyquist's theorem for radiation resistance, though it is often implicitly assumed in the calculation of signal/noise ratio. Llewellyn in 1931 appears to have been the first to recognize the problem, and made the assumption that radiation resistance behaved like an ohmic resistance at atmospheric temperature. F. C. Williams (1937) has analysed the signal/noise ratio for the case of an aerial inductively coupled to a tuned-grid circuit, taking into account valve noise, while K. Fränz (1939) has made a detailed analysis for the case of a generalized coupling. Both writers, however, implicitly adopt Llewellyn's hypothesis, without giving due consideration to its validity.

Seki (1937) made measurements of noise in aerial circuits, tending to confirm the hypothesis, but they are by no means conclusive. Jansky (1937) points out that coupling an antenna to a tuned circuit should lower the latter's thermal noise since the dynamic impedance is reduced, whereas experiment always shows an increase of noise even when atmospheric and man-made interference are absent. Bell (1939) has discussed the problem theoretically and concludes that radiation resistance is not a source of thermal-agitation electromotive force, and that it should be treated as a reactance of equal magnitude when calculating the signal/noise ratio in aerial systems. The present writer does not agree with Bell's arguments, which are discussed below.

§3. AERIAL IN A UNIFORM TEMPERATURE ENCLOSURE

An aerial system will continuously radiate radio-frequency energy by virtue of the fluctuation currents flowing in it arising from its ohmic resistance. It will also receive energy by radiation at the same frequencies from the fluctuation currents in surrounding conductors. Thus fluctuation electromotive forces are induced in the aerial, and these may be attributed to thermal agitation in the radiation resistance, for, as will be shown later, the mean square electromotive force is proportional to the radiation resistance.

If the system is in thermal equilibrium in a uniform temperature enclosure at $T°$ K., then from the Prévost Theory of Exchanges a balance will exist between the energy radiated and that received, and furthermore, this equilibrium will exist at every frequency in the radiation spectrum. At the two terminals of the aerial let the impedance be $Z(=R+jX)$, which includes the ohmic resistance R and the radiation resistance R_r at the frequency ν. The radiation resistance R_r is defined as the ratio of the power radiated by the aerial to the mean square current at the terminals, producing the radiation.

By calculating the rate at which the fluctuations in the aerial radiate energy, and equating this to the power received from the external radiation, the electromotive force induced by the radiation can be found. Let this electromotive force have a mean square value of $d\overline{e_r^2}$ in the frequency band ν to $\nu+d\nu$. By reference to figure 1 it is seen that the fluctuation current flowing in the aerial due to its ohmic resistance R is given by

$$d\overline{i^2}=\frac{4\boldsymbol{k}TR\,.\,d\nu}{Z^2},$$

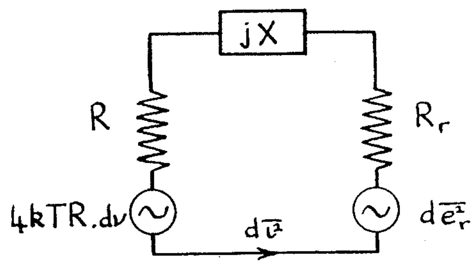

Figure 1. Equivalent circuit of aerial for noise.

and thus R supplies energy which is radiated at the rate

$$d\overline{P_{\text{rad.}}}=R_r\,d\overline{i^2}=\frac{4\boldsymbol{k}TRR_r\,d\nu}{Z^2}.$$

For radiative equilibrium at each frequency, energy must be received at the same rate and supplied to the ohmic resistance R. This received power is

$$d\overline{P_{\text{rec.}}}=\frac{d\overline{e_r^2}}{Z^2}R.$$

For equality of the radiated and received powers it is seen that

$$d\overline{e_r^2}=4\boldsymbol{k}TR_r\,.\,d\nu. \qquad \ldots\ldots(2)$$

The equivalent noise temperature T_r of the radiation resistance is now defined as that temperature which must be ascribed to R_r so as to give the value of the received-noise electromotive force when substituted in Nyquist's equation, i.e. in general such that

$$d\overline{e_r^2}=4\boldsymbol{k}T_rR_r\,.\,d\nu. \qquad \ldots\ldots(3)$$

Thus an aerial in an enclosure at a uniform temperature, T, is in effect the source of a " Nyquist " thermal e.m.f. whose value corresponds to the condition $T_r=T$.

If the aerial temperature is T_a, not equal to T of the enclosure, radiative equilibrium does not exist, and there will be an unbalanced energy flow between the aerial and its surroundings in the sense which tends to equalize T_a and T.

It is concluded that the radiation resistance of an aerial in free space has an equivalent noise temperature of zero; expressed otherwise, there is no other source of radiation present to return energy to the aerial and thus to induce a fluctuation electromotive force. In calculating noise for an aerial in free space we can still use the circuit of figure 1, by putting $d\overline{e_r^2}$ zero. Bell's suggestion of replacing the radiation resistance by a reactance of equal magnitude is incorrect. Radiation resistance always behaves like a resistance, for it is " the ratio of the in-phase electromotive force to the current ", as Bell says. Replacing R_r by jR_r completely alters the impedance of the circuit and thus gives an incorrect value for the noise ; Bell used Williams's formula (1937) for the noise electromotive force, and it was merely necessary to put the temperature of the radiation resistance equal to zero in this formula, and leave the resistance unaltered.

Bell states that an aerial will not radiate energy when it carries fluctuation currents, and uses this argument to deduce that radiation resistance is not a source of thermal electromotive force. If, however, an aerial carries a mean square current $d\overline{i^2}$ in the frequency band $d\nu$, it must radiate energy at the rate $R_r \, . \, d\overline{i^2}$ even though the current is a random one arising from thermal agitation. Bell argues that " the current at any instant is as likely to be in opposition or quadrature as in like phase with the field established by the preceding current element, so that on the whole the work done by the current on the field is zero and no energy is radiated ". It is agreed that the work done on any current element (i.e. electron in motion) by the other electrons in the aerial will on the average be zero, for their field is randomly related to the motion of the electron considered. Work will be done, however, on each electron by its own field: on the classical theory, the power radiated by an electron with acceleration f is

$$P = \frac{2}{3} \frac{e^2 f^2}{c^3},$$

where e = electronic charge,
c = velocity of light *in vacuo*.

The essential difference between radiation from an aerial excited by a generator and one excited by fluctuation currents is that in the first case the electrons move in unison so that mutual radiation effects occur, while in the second only self-radiation by the electrons occurs. Thus in an aerial containing n free electrons, the power radiated in the two cases will be proportional to n^2 and n respectively. An exact analogue is in the scattering of x rays: for the heavier atoms and the longer wave-lengths, coherent scattering occurs and the intensity is proportional to n^2, while for light atoms and medium wave-lengths the intensity of the scattered radiation is proportional to n.

The relation $d\overline{P_{\text{rad.}}} = R_r \,.\, d\overline{i^2}$ thus holds for radiation by fluctuation currents, since the square of the current is also proportional to n and not to n^2, again because the electronic motions are randomly related.

§ 4. THE FREQUENCY DISTRIBUTION OF THE NOISE FIELD

It was seen that in a uniform temperature enclosure, an aerial has an induced noise electromotive force $d\overline{e_r^2}$ of definite value, and it is of interest to calculate the intensity of the radiation field which excites it. The noise field in the enclosure will be perfectly diffuse, i.e. random in direction of propagation and polarization, and a relation between the effective height for diffuse radiation and the radiation resistance of the aerial may be readily deduced by the use of Poynting's theorem. The effective height h_e of the aerial can be defined as the ratio of the potential difference across the terminals to the electric intensity of the radiation field at the aerial, and it will be a function of the polarization and direction of arrival of the wave. The electric and magnetic intensity produced at a distant point by a current in the aerial may be expressed in terms of the current and the effective height of the aerial for the appropriate direction of propagation. The outward energy flow at that point is obtained from Poynting's theorem, and by integrating this over a closed surface surrounding the aerial, the radiation resistance may be derived. Let $\overline{h_e^2}$ be the square of the effective height meaned over all directions of arrival and polarization of the received wave, i.e. the mean square effective height of the aerial for diffuse radiation. Then at a frequency ν, the relation between R_r and $\overline{h_e^2}$ for an aerial in a homogeneous medium of dielectric constant κ and permeability μ is

$$\left.\begin{aligned} R_r &= \frac{8\pi^2\nu^2}{\kappa v^3}\,\overline{h_e^2}\ \text{(e.s.u.)} \\ &= 240\pi^2\sqrt{\frac{\mu}{\kappa}}\,.\,\frac{\overline{h_e^2}}{\lambda^2}\ \text{ohms,} \end{aligned}\right\} \qquad \ldots\ldots(4)$$

where $\lambda = v/\nu$ = wave-length,

v = velocity of electromagnetic waves in the medium $= \dfrac{c}{\sqrt{\kappa\mu}}$.

The mean square electromotive force $d\overline{e_r^2}$ induced in the aerial by the noise field will be related to the mean square electric intensity $d\overline{E^2}$ of the radiation by

$$d\overline{e_r^2} = d\overline{E^2}\,\overline{h_e^2}.$$

Hence from equations (2) and (4)

$$d\overline{E^2} = \frac{32\pi^2 \boldsymbol{k} T\nu^2}{\kappa v^3}\,.\,d\nu.$$

The mean energy density of the radiation in the frequency interval $d\nu$ is thus

$$\left.\begin{aligned} d\overline{\epsilon_\nu} &= \frac{\kappa d\overline{E^2} + \mu d\overline{H^2}}{8\pi} = \frac{\kappa d\overline{E^2}}{4\pi} \\ &= \frac{8\pi \boldsymbol{k} T\nu^2}{v^3}\,.\,d\nu, \end{aligned}\right\} \qquad \ldots\ldots(5)$$

which is the Rayleigh-Jeans distribution law for black-body radiation. Thus the radiation produced by fluctuation currents in an aerial, and the radiation which induces the " Nyquist " noise e.m.f. in an aerial in a uniform temperature enclosure are to be identified with the usual temperature radiation.

The above analysis could have been presented in the reverse order by starting with the Rayleigh-Jeans radiation law, deducing the electric intensity of the radiation, and thus calculating the electromotive force which would be induced in an aerial in a uniform temperature enclosure. This would have been found equal to the value given by Nyquist's theorem for the radiation resistance at the temperature of the enclosure. Furthermore, the existence of a fluctuation electromotive force in ohmic resistances, of the value given by Nyquist's theorem, would be found necessary to give the balance of received and radiated power which occurs for an aerial in thermal equilibrium. This derivation of Nyquist's theorem for ohmic resistance would seem to be free from the objections which Moullin (1938, p. 39) raises in connection with Nyquist's original proof. It is not surprising that Nyquist's theorem and the radiation law should be consistent, for they may both be deduced by counting the number of degrees of freedom of electromagnetic vibration in a given frequency interval. In one case the system considered is one-dimensional, namely, a transmission line, while in the other it is a three-dimensional space.

The quantum theory form of Nyquist's theorem is

$$d\overline{e^2} = \frac{4\boldsymbol{h}\nu R \,.\, d\nu}{e^{h\nu/kT}-1} \qquad \ldots\ldots(6)$$

where $\boldsymbol{h}$ = Planck's constant ($6{\cdot}54 \times 10^{-27}$ erg. sec.). Equation (5) then takes the form of the Planck radiation law,

$$d\overline{\epsilon_\nu} = \frac{8\pi\boldsymbol{h}\nu^3}{v^3(e^{h\nu/kT}-1)} \,.\, d\nu \qquad \ldots\ldots(7)$$

However, the classical and quantum radiation theories are indistinguishable even at high radio-frequencies, for if

$$\nu = 10^8 \quad \text{and} \quad T = 300^\circ \text{ K.},$$

then

$$\boldsymbol{h}\nu/\boldsymbol{k}T = 1{\cdot}6 \times 10^{-5}.$$

This small value of $\boldsymbol{h}\nu/\boldsymbol{k}T$ means that the radio-frequency components of the temperature radiation form only a very small fraction of the total radiation. The total energy density of black-body radiation obtained by integrating (7) from $\nu = 0$ to ∞ is given by the Stefan-Boltzmann fourth-power law:

$$\epsilon = \frac{8\pi^5\boldsymbol{k}^4}{15\boldsymbol{c}^3\boldsymbol{h}^3} T^4 = 7{\cdot}65 \times 10^{-15} T^4 \text{ erg/cm}^3$$

For $T = 300^\circ$ K. this has the value

$$\epsilon = 6{\cdot}2 \times 10^{-5} \text{ erg/cm}^3$$

If the radio-frequency spectrum is taken as being from 0 to 1000 Mc./s. ($\nu=0$ to 10^9), its contribution to the energy density is

$$\Delta\epsilon=\frac{8\pi \boldsymbol{k}T}{c^3}\int_0^{10^9}\nu^2\,.\,d\nu=1\cdot 28\times 10^{-17}\ \text{erg/cm.}^3,$$

or only about $\frac{1}{5\times 10^{12}}$ of the total !

§ 5. THE PRACTICAL PROBLEM

The communication engineer is interested in the problem of the maximum signal/noise ratio obtainable with a given receiving aerial system. The one component of noise e.m.f. which can be calculated with certainty is that due to thermal agitation in the ohmic resistance R of the aerial system, and is given by Nyquist's theorem:

$$d\overline{e^2}=4\boldsymbol{k}TR\,.\,d\nu,$$

where T is the temperature of the aerial.

Unfortunately the discussion above of an aerial in a uniform-temperature enclosure is of little assistance in the computation of the noise e.m.f. in an aerial system erected on the surface of the earth. In the first place, the temperature radiation at the earth's surface is not black-body radiation, nor is the aerial temperature any guide to the actual noise field existing, and secondly, there are other natural (i.e. not man-made) noise fields which can be far in excess of the thermal noise field. If an aerial has an equilibrium temperature T governed purely by radiation of spectral distribution ϵ_ν and $a(\nu, T)$ and $e(\nu, T)$ are its absorptive and emissive powers, the equilibrium equation is

$$\int_0^\infty [a(\nu, T)\epsilon_\nu - e(\nu, T)]\,.\,d\nu=0. \qquad \ldots\ldots(8)$$

Only if ϵ_ν follows a black-body distribution will the integrand be zero at every frequency; in other cases there will be no definite relation between the radiation field at any particular frequency and the equilibrium temperature. This is especially the case for radio frequencies, for as we saw above, the energy density at these frequencies is an extremely small fraction of the total, and hence they play an insignificant part compared with the heat or light frequencies ($\nu=10^{14}$ to 10^{16}) in determining the equilibrium temperature.

In practice, conduction and convection exercise a large influence on the equilibrium temperature of the aerial by tending to equalize it to that of its immediate surroundings. Presumably, the most important source of thermal radiation at the earth's surface is the earth itself, especially in the radio-frequency spectrum, remote from the peak in the energy-distribution curve. Despite the relatively high temperature of the sun (6000° K.), its distance is so great that it cannot contribute appreciably to the noise field on the earth. This is confirmed by Jansky (1935), who states that no solar radiations at radio frequencies can be detected.

The intensity of illumination by starlight is very small compared with that from the zenithal sun, so that stellar thermal noise will be quite undetectable. In a series of elaborate experiments, however, Jansky (1932 to 1937) has established the existence of a noise field whose source is situated in the Milky Way; since we have concluded that thermal noise cannot be responsible, another mechanism must be sought, and G. Reber (1940) has recently suggested an electronic process which accounts for the noise. This interstellar radiation depends upon the directional characteristics and orientation of the aerial, on the time of day and of year, and on ionospheric conditions. Jansky's papers should be consulted for further information, but it may be stated that the noise energy observed on $\lambda = 16{\cdot}7$ m. using a rhombic aerial was always more than 8 db. above (6 times) first-circuit noise, and on occasions reached 31 db. above (1300 times) this level.*

Whatever be the actual value of the noise e.m.f. ($d\overline{e_r^2}$ for a band-width B) induced in the aerial from external sources, it may be taken into account by attributing the appropriate temperature T_r to the radiation resistance R_r:

$$d\overline{e_r^2} = 4\boldsymbol{k}T_r R_r B. \qquad \ldots\ldots(9)$$

T_r will generally be independent of the band-width B (Carson, 1925), but it will be a function of the aerial directivity if the noise radiation is not perfectly diffuse (e.g. interstellar or atmospheric noise). Jansky's measurements have shown that on short waves, even in the absence of atmospherics, T_r is always greater than T, and can exceed 1000 T in unfavourable conditions.

In most receiving systems the aerial is coupled by an intermediate network to a tuned circuit at the grid of the first valve in the receiver, and it is the signal/noise ratio at this point which is required. The evaluation of this ratio may be treated quite generally; let the aerial be connected to the terminals (1, 1) of a linear, passive network at whose terminals (2, 2) it is desired to know the signal/noise ratio (figure 2).

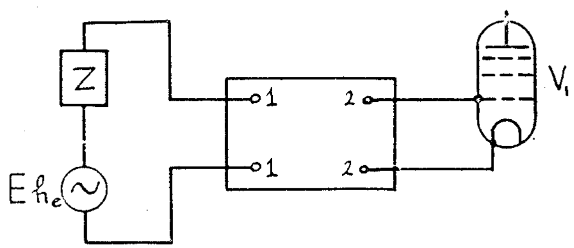

Figure 2. Generalized aerial-grid circuit coupling.

The mean square noise e.m.f. at (2, 2) is given by

$$\overline{e_n^2} = 4\boldsymbol{k}B[n_A^2 T_r R_r + T\Sigma n_s^2 R_s], \qquad \ldots\ldots(10)$$

* Another very variable source of noise radiation is "atmospherics", i.e. radiations produced by electrical disturbances of the atmosphere. These are not amenable to any except a very rough estimation based on previously collected data (Potter, 1931 and 1932).

where R_s = resistive component of the sth impedance element Z_s of the network, excluding the radiation resistance of the aerial;
n_s = modulus of voltage transfer ratio from Z_s to (2, 2);
n_A = modulus of voltage transfer ratio from aerial to (2, 2); and
T = temperature of the ohmic resistances in the network.

Valve noise may be taken into account by adding the term $4\boldsymbol{k}BTR_v$, where R_v is the equivalent noise resistance of the first valve:

$$\overline{e_n^2} = 4\boldsymbol{k}B[n_A^2 T_r R_r + T\Sigma n_s^2 R_s + TR_v]. \qquad \ldots\ldots(11)$$

If the aerial receives a signal of field intensity E, for whose direction of arrival and polarization the effective height is h_e, the signal potential-difference at (2, 2) is

$$e_s = Eh_e n_A.$$

Thus the signal/noise ratio after amplification by the first valve is

$$\rho = \frac{e_s}{\sqrt{\overline{e_n^2}}} = \frac{Eh_e n_A}{\sqrt{4\boldsymbol{k}B[n_A^2 T_r R_r + T\Sigma n_s^2 R_s + TR_v]}} = \frac{\rho_0}{\sqrt{1 + \frac{1}{K}\frac{T}{T_r}}}, \qquad \ldots\ldots(12)$$

where

$$\rho_0 = \frac{Eh_e}{\sqrt{4\boldsymbol{k}BT_r R_r}} = \frac{\text{received signal}}{\text{received noise}}$$

is the absolute maximum of the signal/noise ratio, and

$$K = \frac{n_A^2 R_r}{R_v + \Sigma n_s^2 R_s}$$

is a measure of the overall efficiency of the receiving system comprising the aerial circuit and the first valve. The greater the value of K, the more nearly does ρ approach its limiting value of ρ_0. As an arbitrary criterion, it might be stated that if $K \geqslant 0{\cdot}1$, the system can be considered efficient.

To interpret these results in a practical form, a field strength E_1 can be specified such that the signal/noise ratio is unity after amplification by the first valve. From equation (12)

$$E_1 = E_0 \sqrt{\left(\frac{T_r}{T} + \frac{1}{K}\right)}, \qquad \ldots\ldots(13)$$

where

$$E_0 = \sqrt{(4\boldsymbol{k}TB \,.\, R_r/h_e^2)}. \qquad \ldots\ldots(14)$$

From equation (13) it is seen that the signal/noise ratio reaches its maximum (i.e. E_1 reaches its minimum) when K is a maximum, and the optimum coupling condition for this is clearly independent of T_r/T. Two cases will be considered:

(1) A vertical aerial coupled by a mutual inductance M to the tuned-grid circuit L, R, C (figure 3). If the aerial is not too short, the primary impedance when made non-reactive is practically wholly R_r, and it is then found that

$$K = \frac{r}{1 + R_v(1+r)^2/D}, \qquad \ldots\ldots(15)$$

where $r = \omega^2 M^2/R_r R$ and D = dynamic impedance of the grid circuit with the aerial disconnected.

The optimum value of r is

$$r_{\text{opt.}} = \sqrt{(1 + D/R_v)}, \qquad \ldots\ldots(16)$$

for which K reaches its maximum,

$$K_{\text{max.}} = \tfrac{1}{2}(\sqrt{(1 + D/R_v)} - 1). \qquad \ldots\ldots(17)$$

The optimum coupling is thus always closer than that for maximum signal, viz. $r = 1$.

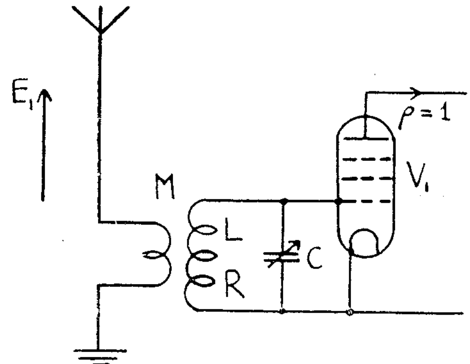

Figure 3. Vertical aerial receiving circuit.

To illustrate these results numerically, consider typical values:

$$B = 5 \text{ kc./s.}, \qquad T = 290^\circ \text{ K.},$$
$$R_v/D = 0{\cdot}1, \qquad \lambda = 20 \text{ m. (15 Mc./s.)}.$$

For a vertically polarized ground-wave E_0 will then have the value 0·0177 μv./m. The variation of E_1 with the coupling r is shown in figure 4 for the cases of

(i) $T_r/T = 0$: Free-space condition, giving absolute minimum of E_1.
(ii) $T_r/T = 1$: Uniform-temperature enclosure at T.
(iii) $T_r/T = 10$: Typical case of low level of received noise.
(iv) $T_r/T = 1000$: Typical case of high level of received noise.

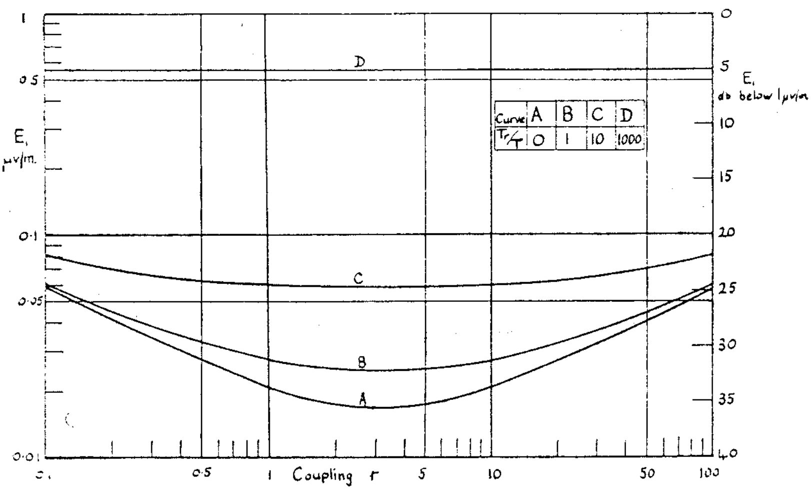

Figure 4. Receiving characteristics of vertical aerial circuit.

From the curves it is concluded that the signal/noise ratio is not critically dependent on the coupling, even for quite low levels of received noise ($T_r/T = 10$).

The curves are representative of a typical case of a high value of K (here ranging from 0·1 to 1·2) which obtains in aerial systems whose radiation resistance is not small compared with the ohmic resistance. In these circumstances the received noise predominates over the circuit and valve noise.

(2) A tuned loop of dynamic impedance D, in which the radiation resistance R_r is small compared with the ohmic resistance R, connected directly in the grid circuit of the first valve. For this case

$$K=\frac{R_r}{R}\cdot\frac{1}{1+R_v/D}\cdot \qquad \text{......(18)}$$

For a loop 0·5 m. square of ohmic resistance 3 ohms, the other parameters being the same as for the vertical aerial case, it is found that $K=0{\cdot}0072$.

The table shows for comparison the values of E_1 for

(*a*) the ideal limiting case of an aerial having no ohmic resistance connected directly to a noiseless valve (i.e. $K=\infty$);

(*b*) the vertical aerial with optimum coupling; and

(*c*) the tuned loop, when its plane is in the position for maximum reception.

It is seen that the loop is a poor receiving system, since in all but high levels of received noise the thermal and valve noise predominate and thus set a relatively high lower limit for E_1. This is typical of aerial systems having a low value of K.

E_1 in μv. /m.

$T_r/T=$	0	1	10	1000
Ideal system ($K=\infty$)	0	0·0177	0·056	0·56
Vertical aerial with optimum coupling ($K=1{\cdot}16$)	0·0163	0·0241	0·058	0·56
Tuned loop ($K=0{\cdot}0072$)	0·208	0·209	0·215	0·66

§ 6. ACKNOWLEDGEMENTS

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REFERENCES

Bakker, C. J. and Heller, G., 1939. "On the Brownian Motion in Electrical Resistances." *Physica*, **6**, 262.

Bell, D. A., 1939. "On the General Validity of Nyquist's Theorem." *Phil. Mag.* **27**, 645.

Bernamont, J., 1937. "Potential Fluctuations at the Ends of a Metallic Conductor of small Volume traversed by a Current." *Ann. Phys., Paris*, **7**, 71.

Carson, J. R., 1925. "Selective Circuits and Static Interference. *Bell Syst. Tech. J.* **4**, 265.

Fränz, K., 1939. "The Limiting Sensitivity in the Reception of Electric Waves, and its Attainability." *Elekt. Nachr.-Tech.* **16**, 92.

Jansky, K. G., 1932. "Directional Studies of Atmospherics at High Frequencies." *Proc. Inst. Radio Engrs, N.Y.*, **20**, 1920 ; 1933. "Electrical Disturbances apparently of Extra-terrestrial Origin." *Ibid.* **21**, 1387 ; 1935. "A Note on the Source of Interstellar Interference." *Ibid.* **23**, 1158 ; 1937. "Minimum Noise Levels obtained on Short-wave Radio-receiving Systems." *Ibid.* **25**, 1517.

Llewellyn, F. B., 1931. "A Rapid Method of Estimating the Signal to Noise Ratio of a High Gain Receiver." *Proc. Inst. Radio Engrs, N.Y.*, **19**, 416.

Moullin, E. B., 1938. *Spontaneous Fluctuations of Voltage* (Oxford University Press).

Nyquist, H., 1928. "Thermal Agitation of Electric Charge in Conductors." *Phys. Rev.* **32**, 110.

Potter, R. K., 1931. "High-frequency Atmospheric Noise." *Proc. Instn. Radio Engrs, N.Y.*, **19**, 1731 ; 1932. "An Estimate of the Frequency Distribution of Atmospheric Noise." *Ibid.* **20**, 1512.

Reber, G., 1940. "Cosmic Static." *Proc. Instn. Radio Engrs, N.Y.*, **28**, 68.

Seki, H., 1937. "Measurement of Receiver-set Noise and Gain in 20 to 55 Mc. Frequencies." *Nippon Elect. Commun. Engr*, 439.

Williams, F. C., 1937. "Thermal Fluctuations in Complex Networks." *J. Instn. Elect. Engrs*, **81**, 751.